A GRAPHICAL METHOD FOR COMPUTING HORIZONTAL TRAJECTORIES IN THE ATMOSPHERE*

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ABSTRACT

Graphical solutions of the horizontal equations of motion are developed for computation of dynamic trajectories. Expedient techniques for using these solutions are discussed. An approximation for the frictional effect is suggested, and graphical application is outlined.

1. INTRODUCTION

The purpose of this paper is (1) to study the quantitative relationships between the changes of the pressure field encountered by a parcel and the changes in its velocity, and (2) to apply the relationships to the construction of horizontal trajectories.

Trajectories of air parcels have been computed by various investigators using the actual winds, geostrophic winds, gradient winds, constant-absolute-vorticity considerations, and combinations of the above. One of the first contributions to a dynamic approach was that of Machta [1]. Recently additional work on "dynamic" computation of trajectories has been published by Takahashi et al. [2], Franceschini and Freeman [3], Wobus [4], the Air Weather Service [5], and Hubert [6]. The methods to be presented in this paper give exactly the same results obtained by Wobus' method, and by the Air Weather Service method for the frictionless case. Franceschini and Freeman's method differs slightly. Hubert's method also differs, his approach being based entirely on hourly forecasts of geopotential or stream function fields by numerical methods. The handling of the frictional or damping terms herein differs considerably from that of Wobus. The other investigators have applied the methods only in the higher troposphere and the stratosphere, while in this paper the methods are applied near the ground.

2. THEORY

The problem is to find the trajectory of a parcel of air in the frictionless atmosphere when its initial velocity is known (or can be closely approximated) and the contour or pressure field is known or can be approximated during a later interval of time. The simplified equations of motion in the atmosphere without friction are

$$\frac{du}{dt}=fv-fv_g=fv' \tag{1}$$

and

$$\frac{dv}{dt}=-fu+fu_g=-fu' \tag{2}$$

where u and v are the east-west and north-south components of the wind, f is the Coriolis parameter, the subscript g denotes the geostrophic wind, and u' and v' denote the geostrophic deviations; i.e., $u'\equiv u-u_g$ and $v'\equiv v-v_g$.

Haurwitz [7] shows these equations have the simple geometric interpretation that the wind is accelerated along a line at right angles to the geostrophic deviation vector $\mathbf{V}'$ and in a right-hand or clockwise sense, as illustrated in figure 1. Note that $\frac{d\mathbf{V}}{dt}$ must always be directed to the right of $\mathbf{V}'$ in the Northern Hemisphere.

For computing the change in the wind during a certain interval of time, one may use the average geostrophic wind on the parcel during that interval, thus assuming that it is constant in magnitude and direction. Then, it follows from equations (1) and (2) that

$$\frac{du'}{dt}=fv' \tag{3}$$

$$\frac{dv'}{dt}=-fu' \tag{4}$$

during the interval. Now, from figure 1

$$u'=V'\cos\theta' \tag{5}$$

$$v'=V'\sin\theta' \tag{6}$$

where $V'=|\mathbf{V}'|=\sqrt{u'^2+v'^2}$. Use of (5) and (6) to transform equations (3) and (4) into a pair for V' and θ' gives

$$\frac{dV'}{dt}=0 \tag{7}$$

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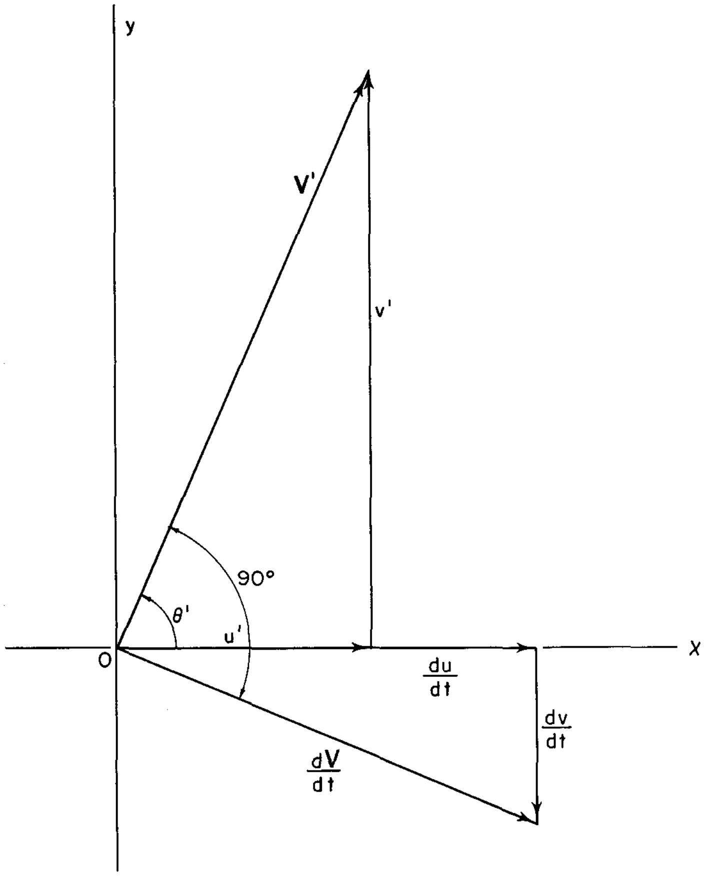

FIGURE 1.—Geometric relationship between the acceleration vector $d\mathbf{V}/dt$ and the geostrophic deviation vector $\mathbf{V}'$.

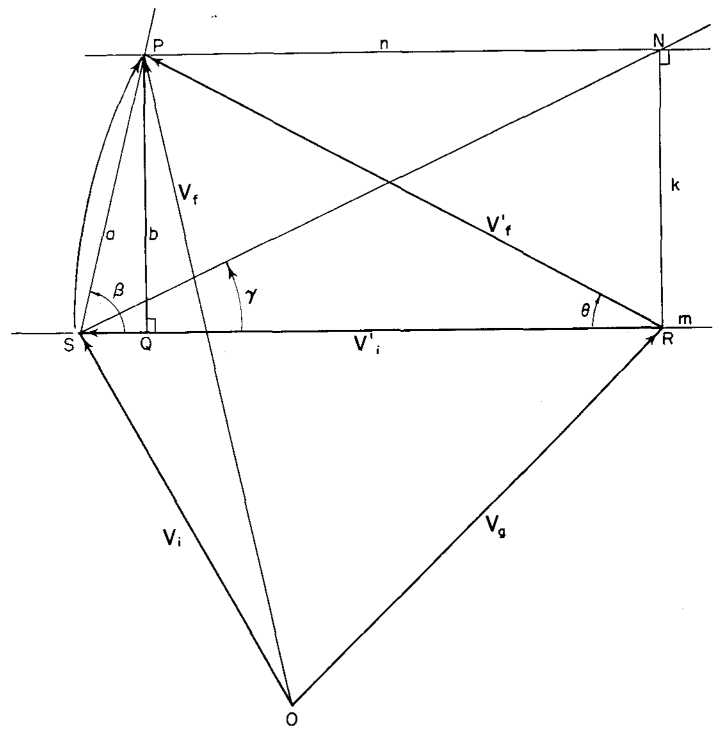

FIGURE 2.—Geometric relationships among the initial wind $\mathbf{V}_i$, the geostrophic wind $\mathbf{V}_g$, the initial geostrophic deviation $\mathbf{V}_i'$, the final geostrophic deviation $\mathbf{V}_f'$, and the final wind $\mathbf{V}_f$.

and

$$\frac{d\theta'}{dt} = -f \tag{8}$$

These equations mean that the *ageostrophic wind vector is constant in magnitude and rotates anticyclonically with a period of one-half a pendulum day.*

Thus, during the time interval in which the geostrophic wind is considered a constant (equal to its average), the change in the wind vector can be represented by the rotation of the ageostrophic wind vector, keeping constant length, clockwise at a rate equal to the Coriolis parameter.

Now, letting $\mathbf{V}_i$=initial wind, and $\mathbf{V}_f$=final wind after time interval τ

$$\mathbf{V}_f = \mathbf{V}_i + f\tau \mathbf{V}' \times \mathbf{k} \tag{9}$$

where the $\mathbf{k}$ is the vertical unit vector.

3. GRAPHICAL SOLUTION

THE ACCELERATION TERM

Since the vector $\mathbf{V}'$ is constant in magnitude, and since $\frac{d\theta'}{dt} = -f$, we may devise a simple graphical solution to evaluate the acceleration term.

In figure 2, $\mathbf{V}_i$, $\mathbf{V}_g$, and $\mathbf{V}_i'$ represent the initial wind, the geostrophic wind, and the initial geostrophic deviation. Then $\mathbf{V}'$ rotates anticyclonically at the rate f. At the end of a time increment τ, a simple rotation with an angle $\theta = f\tau$ gives the final geostrophic deviation $\mathbf{V}_f'$. Then a vector from O to the end point P of $\mathbf{V}_f'$ gives the final wind $\mathbf{V}_f$.

In the application of the relations shown in figure 2 to find the trajectory of a parcel of air during the next hour, the following steps are taken:

(1) Beginning at point O lay off on the weather map a distance OS in nautical miles equal to the observed wind $\mathbf{V}_i$ in knots.

(2) Measure the average geostrophic wind $\mathbf{V}_g$ along the path OS.

(3) Lay off OR in nautical miles equal to $\mathbf{V}_g$ in knots.

(4) With pivot of compass on terminal point R strike an arc SP of angular magnitude $f\tau$ clockwise from terminal point S.

(5) The distance OP in nautical miles equals the final velocity $\mathbf{V}_f$ in knots at the end of one hour.

(6) The trajectory during this first hour is along a vector which is the mean between $\mathbf{V}_i$ and $\mathbf{V}_f$. A simple means of obtaining this average will be treated later.

The striking of the arc in step (4) explains use of the term "arc-strike" as a name for the method.

NOMOGRAM TO AID IN DRAWING TRAJECTORY

Trajectories could be computed as described above by aid of compasses and a straight-edge, but since this involves measuring distances and angles either on the analyzed chart or on a working chart it is a rather slow process. The work can be speeded up by the use of nomograms. In figure 2 it is obvious that because the angle of strike is fixed for any given latitude and time increment and because $|\mathbf{V}_i'|=|\mathbf{V}_f'|$ point P will lie along a line a through point S whose angle β with line m is dependent *only* on the angle of strike, θ. Specifically, since triangle RSP is isosceles

$$\beta=\frac{180^\circ-\theta}{2}. \tag{10}$$

To find the position of P along line a, draw a line n through P parallel to m and construct a perpendicular line k from R to line n. Draw line SN. Line SN helps to locate point P because it can be demonstrated that its angle γ is also dependent *only* on the angle of strike; specifically an examination of figure 2 shows

$$\gamma=\arctan\ (\sin\theta) \tag{11}$$

It is possible then to find point P by knowing merely the positions of the end-points of $\mathbf{V}_i$ and $\mathbf{V}_g$, and the latitude and time interval.

The next item to consider is the average velocity during the time interval involved in drawing the trajectory. The velocity with which the parcel is carried forward should be an integrated average $\mathbf{V}_a$ from the initial velocity $\mathbf{V}_i$ to the final velocity $\mathbf{V}_f$.

If $\mathbf{V}_f$ is not too different from $\mathbf{V}_i$,

$$\mathbf{V}_a\approx\frac{1}{2}\ (\mathbf{V}_i+\mathbf{V}_f) \tag{12}$$

If $\mathbf{V}$ and $\mathbf{V}_i$ are very different, this approximation becomes less accurate and then it is necessary to approach this from a more rigorous standpoint. Wobus [4] does this averaging by means of a nomogram called a Trajectory Computer. Experience dictates that the shorter the time interval used the more accurate the trajectory will be defined—this is particularly so where the contour or pressure fields are changing rather rapidly in time or space. We must look for a system to construct our nomograms rapidly for any time interval and latitude desired.

In figure 3 is reconstructed the portion of figure 2 necessary to aid in finding the average velocity $\mathbf{V}_a$. S and R, which are the terminal points of $\mathbf{V}_i$ and $\mathbf{V}_g$, define $\mathbf{V}'$ at time $t=0$. At some later time t, $\mathbf{V}'$ has rotated through angle θ and point P is then the terminal point of $\mathbf{V}$.

It may be seen from figure 3 (see also equations (5) and (6), noting $\theta=\pi-\theta'$) that

$$u^*=V_i'-V_i\cos\theta \tag{13}$$

$$v^*=V_i'\sin\theta \tag{14}$$

where u^* and v^* are components of the velocity vector $\mathbf{V}^*$ from S to P.

From equation (8) and the relation $\theta=\pi-\theta'$, it follows that $\theta=ft$; thus equations (13) and (14) can be integrated

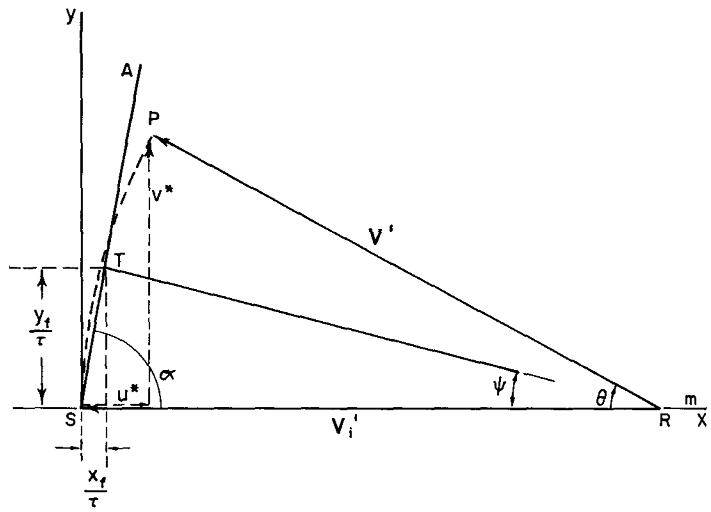

FIGURE 3.—Geometric determination of terminal point T of trajectory from vector relationships shown in figure 2.

for the time interval 0 to τ, giving

$$x_f=V_i'\tau-\frac{V_i'}{f}\sin\theta \tag{15}$$

$$y_f=\frac{V_i'}{f}\ (1-\cos\theta) \tag{16}$$

Since $\mathbf{V}_f-\mathbf{V}_i=\mathbf{V}^*$, equations (15) and (16) give the coordinates of the parcel relative to point S after a time interval τ. If SA is the line through S and the point (x_f, y_f), making the angle α with line m, then from figure 3 and (15) and (16):

$$\tan\alpha=\frac{y_f}{x_f}=\frac{1-\cos\theta}{\theta-\sin\theta} \tag{17}$$

Now since the average velocity $\mathbf{V}_a$ for the particular arc-strike, θ, has the components x_f/τ and y_f/τ, it follows from (17) that the end point of the vector $\mathbf{V}_a$ lies along a line which makes angle α with line m, and α is dependent *only* on the angle of strike; specifically, from (17):

$$\alpha=\arctan\left(\frac{1-\cos\theta}{\theta-\sin\theta}\right) \tag{18}$$

Similarly, by integration of equations (5) and (6), it can be shown that x_f/τ and y_f/τ lie on a line through point R that makes angle $\theta/2$ with line m. Thus in figure 3, $\psi=\theta/2$.

With equations (10), (11), and (18) it is possible to construct quickly a nomogram to compute trajectories at any latitude and for any time period. Example: to construct a computing nomogram for the 30°N. latitude and for intervals of 2 hours. The angle of strike $\theta=ft=4\Omega\sin\phi=30^\circ$ where $\Omega\approx\frac{360^\circ}{24\text{ hr.}}$, the angular speed of the earth's rotation. From equation (10), $\beta=\frac{180^\circ-\theta}{2}=\frac{180^\circ-30^\circ}{2}=75^\circ$. From equation (11), $\gamma=\arctan\sin 30^\circ=26.6^\circ$.

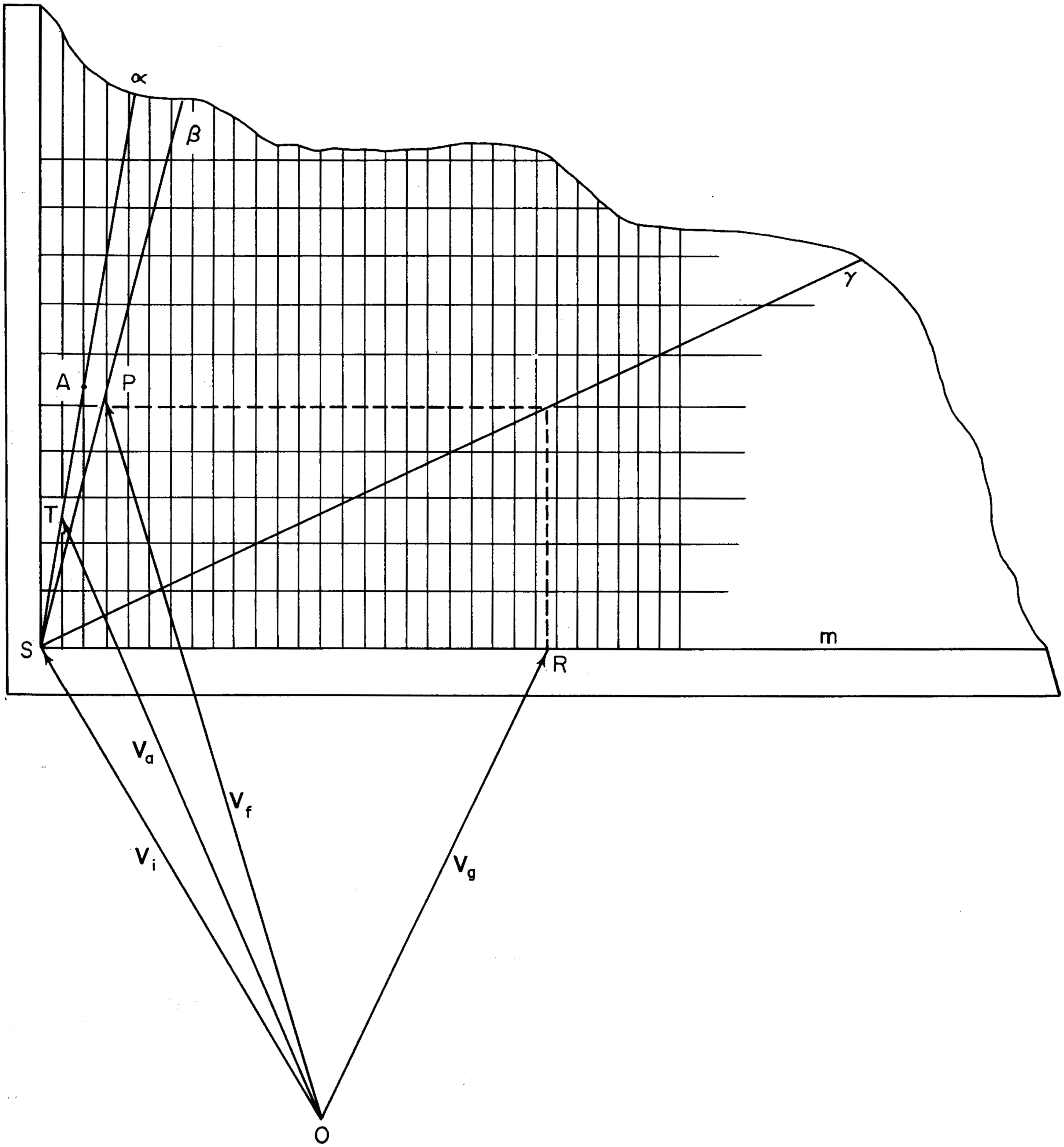

FIGURE 4.—Nomographic determination of terminal point T of trajectory. The lines emanating from point S at angles α, β, and γ are fixed for constant latitude and time interval.

From equation (18), $\alpha = \arctan \dfrac{1-\cos\theta}{\theta-\sin\theta}$

$$= \arctan \frac{1-\cos 30°}{\frac{\pi}{6}-\sin 30°} = 80°.$$

On a piece of graph paper of suitable size draw three lines with angles of 75°, 26.6°, and 80° all emanating from the origin S as in figure 4. For convenience the trajectory is best drawn on transparent paper. Lay this over the map and mark reference points so that the same relative position of paper and chart may be easily found. Mark the point where the trajectory is to be started. (The initial wind here should be known or closely approximated.) From this point lay off a line twice the vector

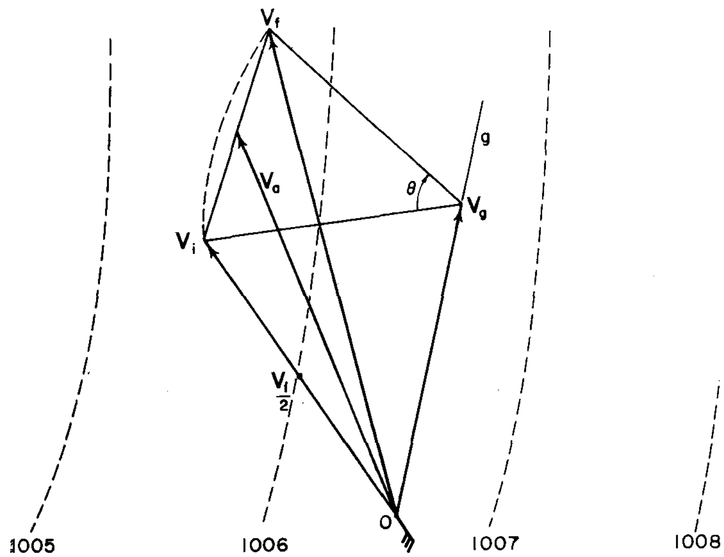

FIGURE 5.—Example of construction of trajectory for model pressure field with observed wind $\mathbf{V}_i$, mean geostrophic wind $\mathbf{V}_g$, angle of arc-strike θ, and final wind $\mathbf{V}_f$. All wind vectors are drawn to lengths corresponding to 3-hr. displacements so that the 3-hr. trajectory is given by the mean wind vector $\mathbf{V}_a$.

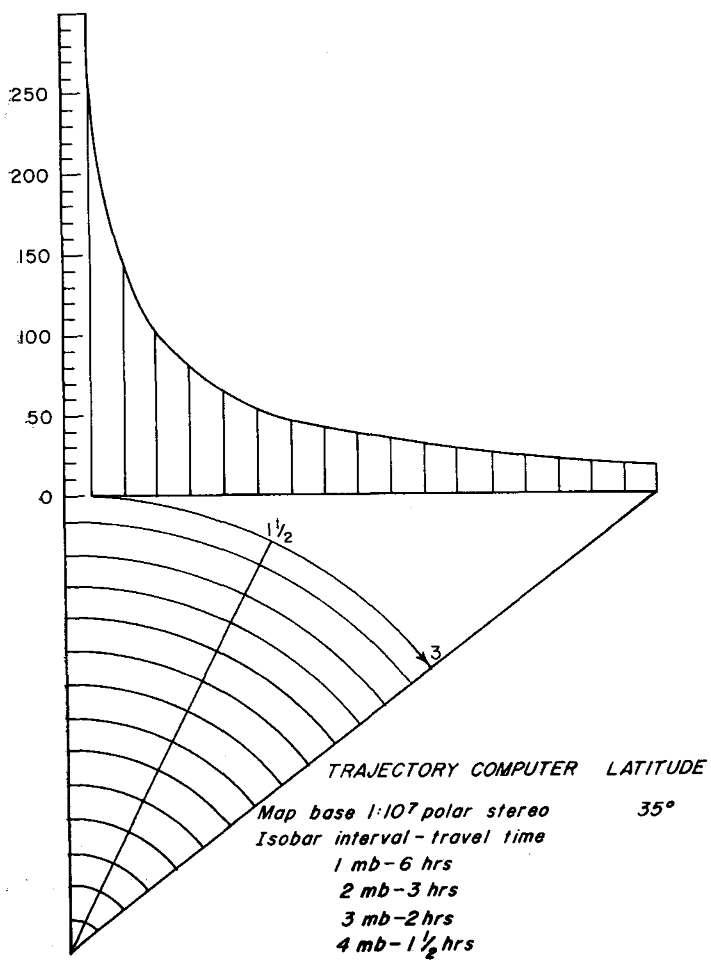

FIGURE 6.—Overlay for rapidly constructing trajectories at latitude 35° for a polar stereographic map base of scale $1:10^7$.

$\mathbf{V}_i$, since the time interval is two hours.

As a first approximation assume this is the actual path taken by the parcel and measure the average geostrophic wind along this path. References [3], [4], and [5] have some good discussions on several methods of ascertaining the average geostrophic wind and the relative merits of these methods.

From the initial point of $\mathbf{V}_i$ lay off twice $\mathbf{V}_g$. Now lay the transparent work-sheet on the computer so that the terminal end, S, of $\mathbf{V}_i$ falls on the origin and the terminal end, R, of $\mathbf{V}_g$ falls along the abscissa. This is shown in figure 4. From R follow the ordinates upward to line γ, then along the abscissa to line β and mark point P. A line from O to P is the final velocity $\mathbf{V}_f$ at the end of 2 hours. With a straight-edge along R and P mark point A on line α and mark a point T halfway from A to S.

The line OT now represents the average velocity $\mathbf{V}_a$ during the first 2 hours, and since it is automatically doubled it is also the trajectory of the parcel for the first 2 hours. $\mathbf{V}_f$ now becomes the initial velocity for the next 2-hour period and the process is repeated.

If the pressure or height gradients are not nearly uniform along and near the path along which the geostrophic average was measured, second and perhaps (rarely) third approximations should be made.

If a second approximation is necessary in the example of the computation of the trajectory for the first 2 hours, place the work-sheet back on the chart and measure the geostrophic wind average along $\mathbf{V}_a$. Lay off the new average $\mathbf{V}_g$ and re-compute $\mathbf{V}_f$ and $\mathbf{V}_a$. *Do not change* $\mathbf{V}_i$, however.

RAPID APPROXIMATE CONSTRUCTION OF TRAJECTORIES*

A method for the rapid construction of frictionless trajectories, based upon equation (12) is presented below. Although the method applies to any latitude or map base, an example is given for 35° N. latitude and a map base of $1:10^7$, polar stereographic projection. Figure 5 is a model pressure pattern showing isobars at a 1-mb. interval. A trajectory is to be constructed at 3-hour intervals beginning at point O. In order to remove partly the effects of surface friction, the so-called "gradient" wind is used as the initial wind at point O. A sheet of transparent paper is placed on top of the map and this wind is projected for 3 hours, or in this case 90 nautical miles, downstream to form the vector $\mathbf{V}_i$.

Next, the mean geostrophic wind direction for the 3-hour period is determined at the midpoint of $\mathbf{V}_i$ (point $\frac{\mathbf{V}_i}{2}$). This same direction is laid off from O to form the line g. The 3-hour mean geostrophic wind speed is measured using a transparent overlay illustrated in figure 6. The upper part of the overlay gives the distance moved by a parcel moving with the mean geostrophic wind. The scale on the left is in nautical miles. The straight and curved lines are spaced at intervals of 20 nautical miles.

*The method was developed by Mr. K. Peterson.

In order to obtain the 3-hour mean geostrophic wind speed, place the scale on the map so that the straight lines on the overlay are parallel to the line g and so that point $\frac{\mathbf{V}_i}{2}$ lies on the base line. The point $\frac{\mathbf{V}_i}{2}$ should be approximately mid-way between the point where an isobar touches the edge of the overlay at zero and the point on the base line 2 mb. to the right. Holding a pencil over the point which is 2 mb. to the right on the baseline, move the scale so that this point is coincident with point O and the straight lines on the scale are parallel to line g. Place a dot at the edge of the scale on line g. This determines the ector $\mathbf{V}_g$, the distance a parcel starting at point O will travel in 3 hours if the wind is geostrophic. If a 6, 2, or 1½-hour travel distance is desired, use the overlay to measure across 1, 3, or 4 mb., respectively. Such an overlay can be prepared for any latitude, map base, map projection, and isobar interval with the aid of a standard geostrophic wind scale and a distance scale. If overlays are prepared for every 5° latitude, the error at latitudes between overlays is negligible.

Place the bottom point of the overlay at point $\mathbf{V}_g$ and let point $\mathbf{V}_i$ touch the left edge of the scale. Using the arcs as a guide, go to the right (anticyclonically) and place a point at the right edge of the scale, locating point $\mathbf{V}_f$. The vector $\mathbf{V}_f$ represents the actual wind after 3 hours. Bisect the chord from $\mathbf{V}_i$ to $\mathbf{V}_f$ to obtain the point $\mathbf{V}_a$. (It can be shown that $\mathbf{V}_a$ should be between the midpoint of the arc and the chord, but the error is negligible). The vector $\mathbf{V}_a$ is the mean actual wind given by equation (12) for the time period; it represents the trajectory for the first 3 hours.

The angle at the bottom of the overlay is obtained from the relation $\theta = f\tau$; here $f\tau = 30°(\sin\phi)\tau$ where ϕ is the latitude and τ is the arc-strike interval. At $\phi = 35°$ and with a 3-hour arc-strike,

$$\theta = 30°(0.574)(3) = 51.8°$$

The above equation can be used to obtain arc-strike angles for any latitude and arc-strike interval.

In order to obtain the next 3-hour trajectory, transfer the transparent paper to the next map, in this case, 3 hours later. Take the vector $\mathbf{V}_f$ and translate it so that the initial point O is coincident with the point $\mathbf{V}_a$. This new vector is used as the actual wind for the next 3 hours. The remainder of the technique is the same as before—draw the mean geostrophic wind direction, use the overlay to determine the mean geostrophic movement, and strike the arc to obtain the new 3-hour final wind and mean wind. This procedure can be repeated indefinitely, but since the trajectories are frictionless, and since their accuracy is dependent on the accuracy with which the geostrophic wind field, both present and future, can be delineated, they must be used with care when projected for much more than 12 hours. While the large-scale features might be forecast with sufficient accuracy, the fine-grain structure might not, and any result dependent on the latter will suffer.

4. FRICTION EFFECT

EFFECT OF FRICTIONAL ACCELERATION ON TRAJECTORIES

Without friction, once a trajectory is started it may be carried on step by step, chain fashion, for as long a period as the contour field is known or can be approximated. However, experience has shown that this tends to give unrealistic results as inertial oscillations dominate the motion.

The introduction of some damping effect will give more reasonable trajectories but will not, of course, eliminate all the difficulties. The major effect of the viscous or frictional forces is to slow down the parcel speeds, and this neglect leads to systematic errors. For this reason, considerable effort has been directed to devising a method of applying frictional corrections. The simplest hypothesis regarding the frictional forces is that they are directed against the wind in direction and are proportional to the first or some higher power of the speed. It is suggested that a closer approximation to reality can be obtained if a correction is made using such a relationship with the constants of proportionality determined statistically from a comparison between trajectories computed by dynamic and kinematic methods for a large number of observations. The suggested procedures are described here; evaluation of the empirical constants will be reported at a later date. Doubtless, the more modern theories of the frictional forces can be incorporated by more elaborate procedures.

THEORY

If the acceleration due to friction of an unspecified nature is defined such that it is proportional to a constant power of the speed, the equations of motion may be written.

$$\frac{du}{dt} = fv' - ku^n$$

$$\frac{dv}{dt} = -fu' - kv^n$$

These equations can be solved graphically if n is known. probably n is a function of the speed itself, but for simplicity a value of 2 was assumed.

The frictional effect, as defined above, was added at the end of each time interval to the computations of the trajectory by the methods already described. This involves finding the time-average of the friction term kV^2 during the interval of time τ in which V_i changes to V_F, where V_F is the final speed after the friction effect has been added.

$$V_F = V_f - \int_0^{\tau} kV^2 dt = V_f - \overline{F}\tau \tag{19}$$

where $\overline{F}$ is average frictional acceleration, V_F is speed at time τ. The average acceleration a during the interval τ is

$$\overline{a} = \frac{V_F - V_i}{\tau} \tag{20}$$

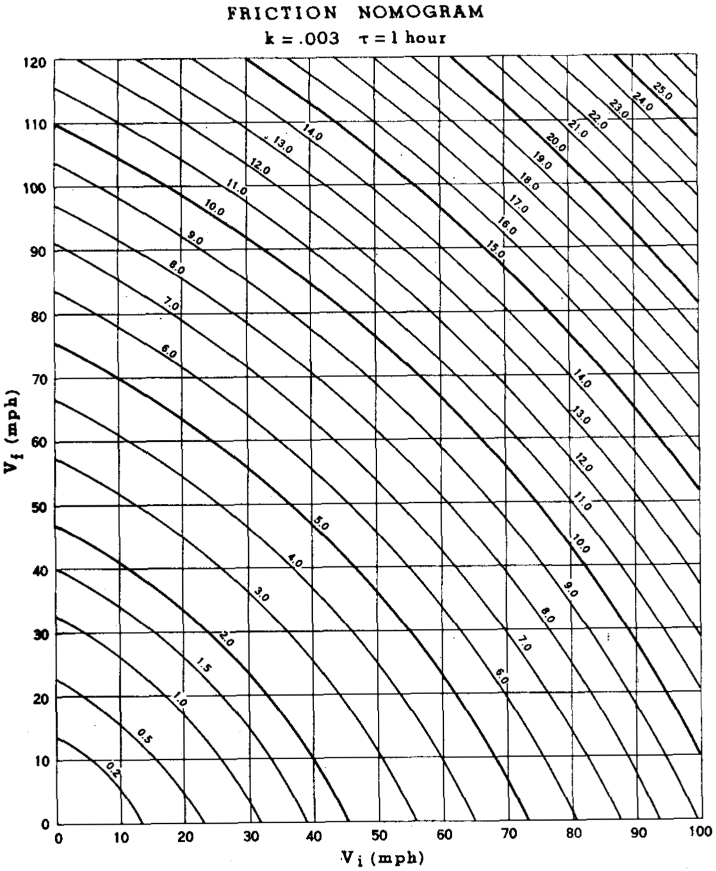

Figure 7.—Friction nomogram based on equation (22) with $k=0.003$, $\tau=1$ hr.

Putting $V=V_i+\bar{a}\,t$ and using equation (20) to eliminate $\bar{a}$, we can evaluate the integral in the middle member of (19). This gives

$$\int_0^\tau kV^2dt=k\tau\left[V_iV_F+\frac{(V_i-V_F)^2}{3}\right] \tag{21}$$

Putting (21) into (19) gives

$$V_F=V_f-k\tau\left[V_iV_F+\frac{(V_i-V_F)^2}{3}\right] \tag{22}$$

GRAPHICAL SOLUTION OF EQUATIONS

Equation (22) can be solved graphically. See figure 7.

In equation (22) $k\tau$ occurs outside the brackets in the last term. This suggests that once a system of isolines of friction deceleration values is calculated this same chart can be used for other values of k and τ by the simple expedient of multiplying the values of the isolines by an appropriate factor. This expedites the testing of different values of k in different situations, either actual or models. To use the friction nomogram, figure 7, enter the abscissa with the value of the initial wind speed V_i and the ordinate with the value of the final wind speed V_f obtained by the arc-strike technique. At the point thus obtained interpolate between curved isolines. This interpolated value is subtracted from V_f to yield V_F, the final speed after reduction due to friction.

APPLICATION TO TRAJECTORIES

As previously mentioned, the friction effect is added stepwise at the end of each trajectory computation. This can be considered only an approximation. Nevertheless, if the method proposed is to be useful, it must be able to predict certain singularities in the field of motion. For example, any wind, initially not in balance, in a straight uniform pressure gradient field will eventually approach a value such that the accelerations on any parcel of air total zero. At this point there is a balance among the forces due to the Coriolis effect, the pressure gradient, and friction.

Furthermore, if the wind is initially considerably out of balance the time required for the wind to adjust to within 5 percent of the theoretical balance velocity is of the order of 10 hours under the conditions assumed. As an actual example, consider a wind with initial components $u=20$ kt. and $v=35$ kt.; the geostrophic wind is a constant at $v_g=40$ kt. and $u_g=0$. For $k=0.003$ mi.$^{-1}$ and at 30° N. latitude, the wind will be in balance when $v=34$ kt. and $u=15$ kt. It takes 26 hours, computed by the methods suggested here, for the wind to adjust to within ± 5 percent of the balance values.

Experiments were made around a hurricane using different values of k. A value of 0.020 gave trajectories that were quite unreasonable in that they headed more directly across the isobars than is observed. With a value of 0.0003 the winds acquired speeds that were much higher than observed and overshot the isobars leading to considerable motion toward higher pressure. A value of 0.003 gave trajectory winds more nearly like those observed.

RESULT OF FRICTIONAL ACCELERATION

The fact that adding friction always accelerates the air parcel toward a certain balance value leads to an interesting corollary. This is that if several winds of different values (both in speed and direction) are all subjected to the same geostrophic field and this field is changing with time the different winds will approach the same value because of frictional acceleration. This is not necessarily the value where balance requirements are satisfied since the geostrophic wind itself may be changing.

The results of a test of this corollary are shown graphically in figure 8. The four initial winds, A, B, C, and D, are shown at the bottom left of the figure. After 6 hours in a geostrophic wind field which appears as at upper center, their values became as at the lower center, and after the next 6 hours with geostrophic wind as at upper right the final values are as at lower right. Note that at the end of each of the two 6-hour intervals the differences among the several winds are progressively reduced.

Trajectories in several hurricanes were computed by this method, using actual data for pressure fields. The results are encouraging. The use of $k=0.003$ mi.$^{-1}$ yielded trajectories that in most respects are similar to trajectories drawn from actual wind data where available. These

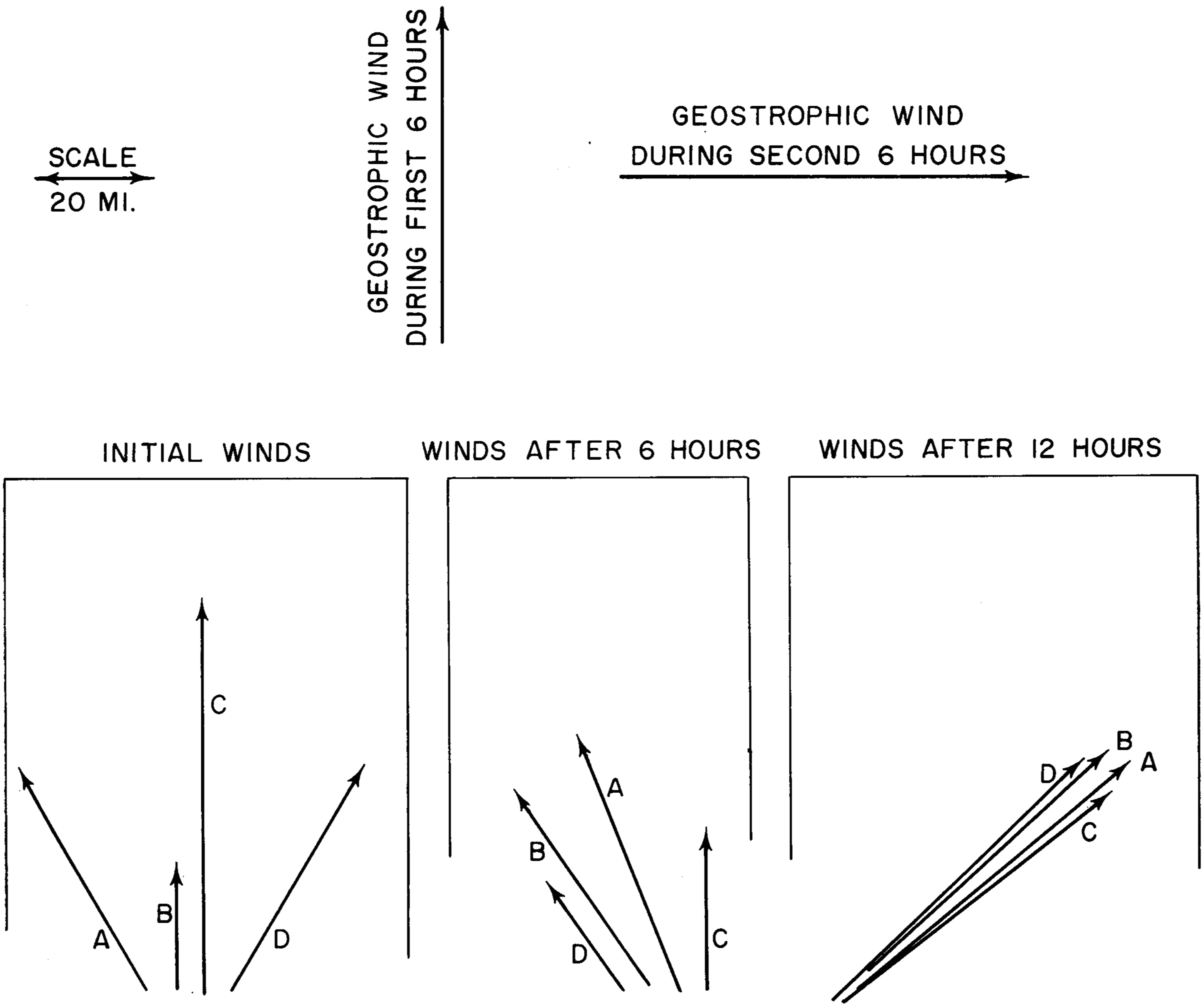

FIGURE 8.—Illustration of tendency of different initial winds subjected to the same changing pressure field to approach the same value because of frictional acceleration.

trajectories together with evaluations of k itself will be discussed at greater length in a subsequent article.

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